



Happy New Year!..

Religious or no, the end of the festive season sees many of us turn our attention to the dreaded double D's – diets and debts. I'm not sure I am a good example for the former, so we thought we'd look at the latter in our first newsletter of 2015. Just how are we going to pay everything off..?

All the best for 2015.

Liam Bastick, Managing Director, SumProduct



Modelling Debt

This month we look at the topical subject of debt. Our examples below include the seemingly more convoluted mathematical formulae which arrive at the same answer. This is because some of the useful functions (e.g. **CUMPRINC**) are not in the "basic" Excel function directory and will

not be recognised without the Analysis ToolPak add-in installed in some versions of Excel (use ALT + T + I to check whether this add-in is installed). The mathematical alternatives work without relying upon this add-in using no more than the standard operators plus the LOG function on occasion.

Problem 1: Minimum Payment Calculations (PAiN Relief)

This is often referred to as the mortgage calculator. Here, the aim is to calculate what the regular repayment is per period to service and pay off a debt over a given amount of time.

For example, if I borrow \$300,000 over 25 years at an interest rate of 12% p.a. what will my regular monthly payments be (assuming no change of rate)?

The answer to this question is given by the formula:

$$P = Ai / (1 - (1 + i)^{-N}) \text{ where:}$$

- **P** = regular periodic payment
- **A** = amount borrowed
- **i** = periodic interest rate
- **N** = total number of repayment periods

(Interesting that the acronym for remembering the mortgage variables is **PAiN!**)

In our example, crunching the numbers (using a periodic interest rate of 1%, being 12% / 12 and total number of periods being 25 x 12 = 300) gives a monthly repayment of \$3,159.67.

The same calculation could have been performed in seconds using Excel's built-in **PMT** function:

PMT(rate,number_of_periods,amount_of_loan)

It should be noted that using PMT will give the same solution, but be negative instead. This is because Excel's financial functions distinguish between cash inflows (positive) and outflows (negative).

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Minimum Payment Calculation - PMT											
2		Debt Payment Examples											
3		Go to Table of Contents											
4		← →											
5													
6													
7		Minimum Payment Calculation - PMT											
8													
9		Payments in Arrears											
10													
11		Imagine you borrow an amount A to be repaid over N periods with a periodic interest rate of i .											
12		Assuming payments are made at the end of each period, the amount to be paid each period will be P :											
13													
14		$P = Ai / (1 - (1 + i)^{-N})$											
15													
16													
17		Illustration											
18													
19		This formula shows how it works:											
20													
21													
22		Amount (A)											
23		Repayment Period (Years)											
24		Repayment Period (Months) (N)											
25		Annual Interest Rate (%)											
26		Monthly Interest Rate (%) (i)											
27		Monthly Payment											
28		PMT Calculation											
29													
30													

Problem 2: Calculating the Outstanding Balance

If we are looking to re-finance or pay off a loan, we need to be able to forecast the balance outstanding at a point of time (ignoring penalties, etc.).

Continuing the example from above, I have borrowed \$300,000 over 25 years at an interest rate of 12% p.a. and I am making my monthly (minimum) repayments. After three years, I am able to re-finance my loan without incurring any penalties. What would be the outstanding amount at this point in time?

The answer to this question is given by the formula:

$$B = A(1+i)^n - (P((1+i)^n - 1) / i) \text{ where:}$$

- **B** = balance outstanding
- **P** = regular periodic payment
- **A** = amount borrowed
- **i** = periodic interest rate
- **n** = number of periods payments have been made ($n \leq N$)

Using the same periodic interest rate of 1%, the monthly repayment of \$3,159.67 (calculated above) and using 36 (12 x 3) periods for n , the amount outstanding would be \$293,121.81, i.e. only \$6,878.19 (principal) has been paid off. More scarily, if we calculate the total of the payments made (36 x \$3,159.67), then the inferred interest is a whopping \$106,870.02! Best not to think too long about it...

The same calculation could have been performed quickly using Excel's **CUMPRINC** function:

CUMPRINC(rate,life_of_loan,amount_of_loan,start_period,end_period,type)

This requires explanatory notes:

- Again, the **rate** is the periodic rate (here 1% not 12%)
- **life_of_loan** is the total length of the loan, not just to date. This is needed to calculate implicitly the regular repayment amount
- **CUMPRINC** actually calculates the amount of principal paid off for an interval of time (**CUMPRINC** = cumulative principal). Therefore, the start and end dates must be stipulated, and if we are calculating the aggregate from the beginning, the **start_date** must be 1 (first period)
- The **type** determines whether payments are made in arrears (type 0) or in advance (type 1)
- To calculate the balance outstanding using this method we simply compute **Amount + CUMPRINC calculation**

It should be noted that, similar to PMT, CUMPRINC gives a negative value for similar reasons. This is why the formula employs addition rather than subtraction.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1		Remaining Loan Balance - CUMPRINC												
2		Debt Payment Examples												
3		Go to Table of Contents												
4		← →												
5														
6														
7		Remaining Loan Balance - CUMPRINC												
8														
9		Payments in Arrears												
10														
11		Imagine you borrow an amount A to be repaid over N periods with a periodic interest rate of i .												
12		Assuming payments are made at the end of each period, the remaining loan balance B after n periods (n ≤ N) will be:												
13														
14		$B = A(1+i)^n - ((P/i)((1+i)^n - 1))$												
15														
16														
17		Illustration												
18														
19		This formula shows how to calculate it:												
20														
21		Amount (A)	\$300,000											
22		Repayment Period (Years)	25											
23		Repayment Period (Months) (N)	300											
24		Month Number (n ≤ N)	36											
25		Annual Interest Rate (%)	12.00%											
26		Monthly Interest Rate (%) (i)	1.00%											
27		Monthly Payment (P)	\$3,159.67 using $P = Ai / (1 - (1+i)^{-N})$											
28														
29		Outstanding Balance (B)	\$293,121.81											
30														
31		CUMPRINC Calculation	\$293,121.81											
32														
33		CUMPRINC actually calculates the principal paid off between periods x and y , where y ≥ x .												
34		This is why the value '1' has been hardcoded into the formula above.												
35		Therefore, the balance remaining is the original amount less the principal paid off.												
36														
37		Total Interest Paid so far	\$106,870.02											
38														

Problem 3: How Long Before the Debt is Paid Off?

What if we decide to turn the problem around? Instead of deciding how long we want to take to pay off the debt, how do we calculate how long a debt will take to pay off if we decide to stipulate what the regular payment will be each month instead?

Returning to my example, if I borrow \$300,000 at an interest rate of 12% p.a. how long will it take to pay off if I pay \$5,000 each month? This does assume that our payment exceeds the accrued interest each month (here, \$300,000 x 1% = \$3,000).

The answer to this question is given by the formula:

$$N = -\log(1 - (Ai / P)) / \log(1 + i) \text{ where:}$$

- **N** = total number of repayment periods
- **A** = amount borrowed
- **i** = periodic interest rate
- **P** = regular periodic payment

(The logarithmic base is irrelevant as long as the same one is used for both the numerator and the denominator of the above quotient.)

Plugging the inputs into the formula gives us a value of 92.09, i.e. 93 periods (fractions of periods do not really make sense here).

The same calculation could have been computed simply using the **NPER** function:

NPER(rate,-payment,amount_of_loan)

Note that the payment must be negative and the amount of the loan must be positive.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1		Number of Periods to Pay Off a Loan - NPER																
2		Debt Payment Examples																
3		Go to Table of Contents																
4																		
5																		
6																		
7		Number of Periods to Pay Off a Loan - NPER																
8		Payments in Arrears																
9																		
10		Imagine you borrow an amount A with an associated periodic interest rate of i .																
11		How many periods will it take to pay off the loan if you pay an amount P at the end of each period?																
12																		
13		$N = -\log(1 - (Ai / P)) / \log(1 + i)$ <small>Note: it does not matter which logarithmic base you use as long as it is the same for the numerator and the denominator</small>																
14																		
15																		
16																		
17		Illustration																
18																		
19		This formula shows how it works:																
20																		
21		Amount (A) <input type="text" value="\$300,000"/>																
22		Annual Interest Rate (%) <input type="text" value="12.00%"/>																
23		Monthly Interest Rate (%) (i) <input type="text" value="1.00%"/>																
24		Monthly Payment (P) <input type="text" value="\$5,000"/>																
25																		
26		Number of Months (N) <input type="text" value="92.09"/> <small>This figure could be rounded up though if not an integer</small>																
27																		
28		NPER Calculation <input type="text" value="92.09"/>																
29																		
30																		
31		Payments in Advance																
32																		
33		Again, you borrow an amount A with an associated periodic interest rate of i .																
34		How many periods will it take to pay off the loan if you pay an amount P at the beginning of each period?																
35																		
36		$N = 1 - \log(1 - ((A - P) / P)) / \log(1 + i)$ <small>Note: it does not matter which logarithmic base you use as long as it is the same for the numerator and the denominator</small>																
37																		

Further Reading

Much of the above is reproduced from our previous article on debt: **A Debt to Repay**. Check out www.sumproduct.com/thought for further details, including an Excel example file illustrating the examples above that you may download freely.

Reader's Question

I have a cell in Excel that displays "#####" no matter how wide I make the column. What's going on..?

	A	B	C
1	#####		
2			
3			

Usually, this occurs when a numerical value cannot be displayed in the cell due to the width of the cell being insufficient. Widening the column usually fixes the issue.

However, if Excel continues to display a string of # symbols, no matter how wide you make your column, it is likely that Excel is attempting to display the contents of the cell as a date or time, but the cell contains a negative value.

Dates and times are stored as positive serial numbers within Excel. Therefore, a negative value represents an invalid date or time. Excel displays this by filling the cell with # symbols no matter what the width may be (as here).

If the cell is meant to contain a date or time, amend the formula to prevent negative outcomes. If the cell is not meant to contain a date or time, change the formatting of the cell (**CTRL + 1**).



Upcoming SumProduct Training Courses

Dubai 21st - 23rd Mar 2015: Strategic Planning, Forecasting and Budgeting
Hong Kong 5th - 8th May 2015: Strategic Planning, Forecasting and Budgeting
London 18th - 20th May 2015: Strategic Planning, Forecasting and Budgeting
Singapore 26th - 29th May 2015: Strategic Planning, Forecasting and Budgeting
New York 31st Aug - 2nd Sep 2015: Strategic Planning, Forecasting and Budgeting
London 19th - 21st Oct 2015: Strategic Planning, Forecasting and Budgeting
Hong Kong 26th - 29th Oct 2015: Strategic Planning, Forecasting and Budgeting
Dubai 8th - 10th Dec 2015: Strategic Planning, Forecasting and Budgeting

Key Strokes

Each newsletter, we'd like to introduce you to several useful keystrokes you may or may not be aware of. This month we thought we would provide a comprehensive list of function key shortcuts:

Keystroke	What it does
F1	Activate Help
F2	Toggle Select, Edit and Enter / Point modes
F3	Paste Names
F4	Redo / Edit (F2) Mode: Toggle \$ references
F5	Go To / Refresh File List
F6	Next Pane
F7	Check Spelling
F8	Extend Selection Mode
F9	Calculate Now
F10	Activate Menus
F11	Insert Chart on New Sheet
F12	Save As
F13	Creates imaginary keyboard

There are over 540 keyboard shortcuts in Excel. For a comprehensive list, please download our Excel file at <http://www.sumproduct.com/thought/keyboard-shortcuts>.



Coming Soon

We thought we would put our readers on advanced notice that SumProduct is presently working on video content for the www.sumproduct.com website, including online training. More details to follow, but "watch this space". In the meantime, if you have suggested topics for this new offering (or would like to offer potential Excel content – no "marketing opportunities" please) drop us a line at contact@sumproduct.com.

Our Services

We have undertaken a vast array of assignments over the years, including:

- Business planning
- Building three-way integrated financial statement projections
- Independent expert reviews
- Key driver analysis
- Model reviews / audits for internal and external purposes
- M&A work
- Model scoping
- Project finance
- Real options analysis
- Refinancing / restructuring
- Strategic modelling
- Valuations
- Working capital management

If you require modelling assistance of any kind, please do not hesitate to contact us at contact@sumproduct.com.

Convert others!

These newsletters are not intended to be closely guarded secrets. Please feel free to forward this newsletter to anyone you think might be interested in converting to "the SumProduct way".

If you have received a forwarded newsletter and would like to receive future editions automatically, please subscribe by completing our newsletter registration process found at the foot of any any www.sumproduct.com web page.

Any Questions?

If you have any tips, comments or queries for future newsletters, we'd be delighted to hear from you. Please drop us a line at newsletter@sumproduct.com.

Training

SumProduct offers a wide range of training courses, aimed at finance professionals and budding Excel experts. Courses include Excel Tricks & Tips, Financial Modelling 101, Introduction to Forecasting and M&A Modelling.

Check out our more popular courses in our training brochure:



Drop us a line at training@sumproduct.com for a copy of the brochure or download it directly from <http://www.sumproduct.com/training>.

Registered Address: SumProduct Pty Ltd, Level 6, 468 St Kilda Road, Melbourne VIC 3004
Melbourne Address: SumProduct Pty Ltd, Level 9, 440 Collins Street, Melbourne VIC 3000

contact@sumproduct.com | www.sumproduct.com | +61 3 9020 2071